

Quantum control of entanglement and information of two solid state qubits: remote control of dephasing

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We investigate the scheme for controlling information characterized by Von-Neumann entropy and the stationary state entanglement characterized by concurrence of two solid state qubits in the collective dephasing channel. It is shown that the local maximal value of the stationary state concurrence always corresponds to the local minimal value of information. We also propose a scheme for remotely controlling the entanglement of two solid state qubits against the collective dephasing. This idea may open a door to remotely suppress the detrimental effects of decoherence.

PACS numbers: 03.65.Ud, 03.67.-a, 05.40.Ca

I. INTRODUCTION

Quantum entanglement plays an important role in quantum information processes, which can exhibit the nature of a nonlocal correlation between quantum systems that have no classical interpretation [1]. Recently, it has been recognized that entanglement can be used as an important resource for quantum teleportation or quantum computation. Ordinarily, entanglement can be destroyed by the interaction between quantum systems of interest and its surrounding environments. Certain kind of the interaction between the physical system and environments or measuring apparatus can lead to the collective dephasing, which occurs in the physical systems such as trapped ions, quantum dots. Collective dephasing allows the existence of the so-called decoherence-free subspace [2]. Several strategies have been proposed to suppress the detrimental effects of decoherence, while at the same time allowing for robust manipulation of the quantum information [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25], for example, the quantum error correction schemes [15, 16, 17, 18, 19, 20], feedback implementations [21], quantum error avoiding approach [22, 23], dynamical decoupling techniques [24], engineering of pointer state methods [25]. Here, we propose a scheme for remotely controlling the dephasing of two qubits. It is shown that, via a preexistent multipartite entanglement and the quantum erasing process, we can remotely control the entanglement of two qubits in the collective dephasing channel.

Recently, the quantum information processes in the presence of the collective dephasing have intrigued much attention [3, 4, 7, 8]. Khodjasteh and Lidar have investigated the universal fault-tolerant quantum computation in the presence of spontaneous emission and collective dephasing [3]. Hill and Goan have studied the effect

of dephasing on proposed quantum gates for the solid-state Kane quantum computing architecture [4]. It has been shown that some kinds of entangled states of two qubits are very fragile in the presence of collective dephasing and the others are very robust [7]. So, it is very important to protect the fragile entangled states from completely losing their entanglement. Here, we comparatively investigate the information characterized by Von-Neumann entropy and the stationary state entanglement quantified by concurrence of two locally driven qubits in the presence of collective dephasing. We show that one can transform the fragile entangled states into the stationary entangled states under the collective dephasing by making use of a finite-time external driving field. The local maximal value of the stationary state concurrence corresponds to the local minimal value of information.

The disentanglement of entangled states of qubits is also a very important issue for quantum information processes, such as the solid state quantum computation. For example, in quantum registers, some kinds of undesirable entanglement between the qubits can lead to the decoherence of the qubit [6]. Yu and Eberly have found that the time for decay of the qubit entanglement can be significantly shorter than the time for local dephasing of the individual qubits [7, 8]. The collective dephasing can be described by the master equation

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{\gamma}{2} (2\hat{J}_z \hat{\rho} \hat{J}_z - \hat{J}_z^2 \hat{\rho} - \hat{\rho} \hat{J}_z^2), \quad (1)$$

where γ is the decay rate. \hat{J}_z are the collective spin operator defined by

$$\hat{J}_z = \sum_{i=1}^2 \hat{\sigma}_z^{(i)} / 2, \quad (2)$$

where $\hat{\sigma}_z$ for each qubit is defined by $\hat{\sigma}_z = |1\rangle\langle 1| - |0\rangle\langle 0|$. Previous studies have shown that two of the four Bell states $|\Psi^\pm\rangle \equiv \frac{\sqrt{2}}{2}(|11\rangle \pm |00\rangle)$ are fragile states in the collective dephasing channel, while the others Bell states $|\Phi^\pm\rangle \equiv \frac{\sqrt{2}}{2}(|10\rangle \pm |01\rangle)$ are robust entangled states.

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This paper is organized as follows: In section II, study the system in which one of two solid state qubits a collective dephasing environment is driven by a finite time external field. We compare the information characterized by Von-Neumann entropy and the entanglement characterized by concurrence of the stationary state and find that the local maximal value of the stationary state concurrence always corresponds to the local minimal value of information. In section III, we investigate how to remotely control the dephasing of two qubits making use of a preexistent multi-partite entanglement and the quantum erasing process. In section IV, there are some conclusions.

II. INFORMATION AND ENTANGLEMENT IN THE STATIONARY STATE OF TWO SOLID STATE QUBITS

Here, we investigate the model in which two solid state qubits are exposed in a collective dephasing channel and one of two qubits is simultaneously driven by a finite time external field. The dynamics of two qubits can be described by the following master equation

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{2}[\Omega_1(t)\hat{\sigma}_x^{(1)}, \hat{\rho}] + \frac{\gamma}{2}(2\hat{J}_z\hat{\rho}\hat{J}_z - \hat{J}_z^2\hat{\rho} - \hat{\rho}\hat{J}_z^2),$$

where $\Omega_1(t) = \Omega_1\Theta(T-t)$ is the intensity of the time dependent external driving field acted on the qubit 1, $\Theta(x)$ is the unit step function and equals one for $x \geq 0$ and equals zero for $x < 0$. $\sigma_x^{(1)} \equiv |1\rangle_1\langle 0| + |0\rangle_1\langle 1|$ is one of the Pauli matrices. In the following calculations, we will show that the action time T of the external driving field plays a special role in the stationary state entanglement of two qubits. After numerically solving the master equation (3), we can obtain the stationary state density matrix of two qubits. Without loss of generality, the stationary state density matrix has the form

$$\begin{aligned} \rho_s = & a(\frac{\Omega_1}{\gamma}, \gamma T)|11\rangle\langle 11| + b(\frac{\Omega_1}{\gamma}, \gamma T)|10\rangle\langle 10| \\ & + c(\frac{\Omega_1}{\gamma}, \gamma T)|01\rangle\langle 01| + d(\frac{\Omega_1}{\gamma}, \gamma T)|00\rangle\langle 00| \\ & + f(\frac{\Omega_1}{\gamma}, \gamma T)|10\rangle\langle 01| + f^*(\frac{\Omega_1}{\gamma}, \gamma T)|01\rangle\langle 10|. \end{aligned}$$

In order to quantify the degree of entanglement, we adopt the concurrence C defined by Wootters [9]. The concurrence varies from $C = 0$ for an unentangled state to $C = 1$ for a maximally entangled state. For two qubits, in the "Standard" eigenbasis: $|1\rangle \equiv |11\rangle$, $|2\rangle \equiv |10\rangle$, $|3\rangle \equiv |01\rangle$, $|4\rangle \equiv |00\rangle$, the concurrence may be calculated explicitly from the following:

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \quad (5)$$

where the $\lambda_i (i = 1, 2, 3, 4)$ are the square roots of the eigenvalues in decreasing order of magnitude of the "spin-flipped" density matrix operator $R = \rho_s(\sigma^y \otimes \sigma^y)\rho_s^*(\sigma^y \otimes \sigma^y)$

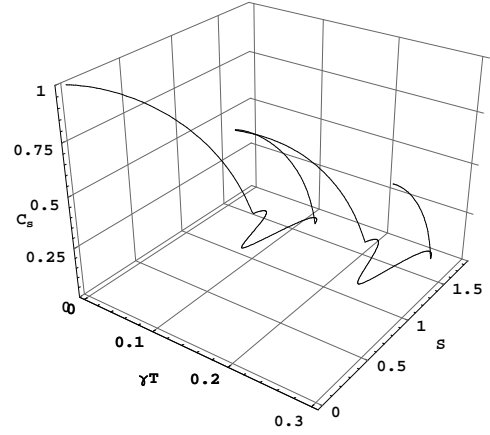


FIG. 1: The stationary state concurrence C_s and the information S of the stationary state are plotted as the function of the parameter γT with $\frac{\Omega_1}{\gamma} = 41.25$. In this case, two qubits are initially in the Bell state $|\Phi^-\rangle$.

σ^y), where the asterisk indicates complex conjugation. The concurrence related to the density matrix ρ_s can be written as

$$C_s = 2 \max[0, |f(\frac{\Omega_1}{\gamma}, \gamma T)| - \sqrt{a(\frac{\Omega_1}{\gamma}, \gamma T)d(\frac{\Omega_1}{\gamma}, \gamma T)}]. \quad (6)$$

The information quantified by the Von-Neumann entropy of the stationary state of two qubits is defined by $S = -\text{Tr}(\rho_s \log_2 \rho_s)$. We can easily obtain the analytical expression of the information S as follows,

$$\begin{aligned} S = & -a \log_2 a - d \log_2 d \\ & -\beta_+ \log_2 \beta_+ - \beta_- \log_2 \beta_-, \end{aligned} \quad (7)$$

where

$$\beta_{\pm} = \frac{b + c \pm \sqrt{(b-c)^2 + 4|f|^2}}{2}. \quad (8)$$

Firstly, we consider the case in which two qubits are initially in the Bell state $|\Phi^-\rangle$, i.e., the Bell singlet state.

In Fig.1, the stationary state concurrence and the information of the stationary state are plotted as the function of the parameter γT . It is shown that the values of the scaled action time γT which locally maximizes the stationary state entanglement always locally minimizes the information. Both the information of the stationary state and the stationary state concurrence oscillate with γT , which imply one can control both the entanglement and information of the stationary state of two qubits in the collective dephasing environment by adjusting the scaled action time γT of the locally driving field. Then, we consider the case in which two qubits are initially in the standard Werner state. The standard two-qubit Werner state was defined by [5]

$$\rho_W = r|\Phi^-\rangle\langle\Phi^-| + \frac{1-r}{4}I \otimes I, \quad (9)$$

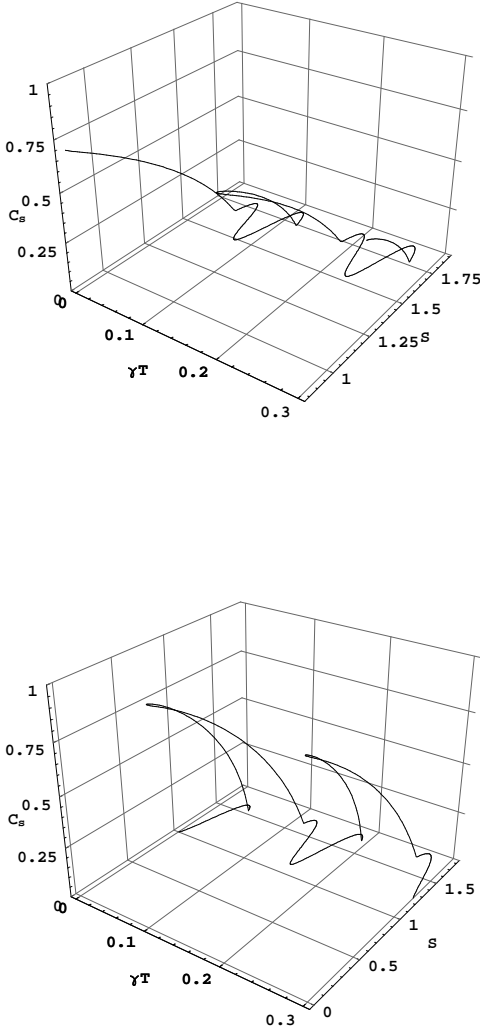


FIG. 3: The stationary state concurrence C_s and the information S of the stationary state are plotted as the function of the parameter γT with $\frac{\Omega_1}{\gamma} = 41.25$. In this case, two qubits are initially in the Bell state $|\Psi^+\rangle$.

where $r \in [0, 1]$, and I is the identity operator of a single qubit. The concurrence of the standard Werner state decreases with its information. In Fig.2, we display the stationary state concurrence and the information of the stationary state as the function of the parameter γT . Two qubits are initially in $0.8|\Phi^-\rangle\langle\Phi^-| + 0.05I \otimes I$. We can see that the initial mixedness changes the stationary state concurrence and information even though Ω_1/γ and γT are both fixed. While the correspondence between the local maximal value of the concurrence and the local minimal value of the information is also valid.

At the end of this section, we consider the case in which two qubits are initially in the fragile entangled state $|\Psi^+\rangle$. From Fig.3, we know that two qubits is separable if γT equals zero, which is different from the case with the initial robust entangled state $|\Phi^-\rangle$. However, the correspondence between the local maximal value of the concurrence and the local minimal value of the in-

formation is valid again. From Figs.1-3, we can observe that improving the stationary state entanglement of two qubits implies the decrease of the information and not vice versa. The correspondence between the entanglement and information entropy of the stationary state may help us to experimentally quantify the entanglement of two qubits. In Ref.[26], a scheme for direct estimations of the functionals of the density matrix has been proposed. In their scheme, one can extract certain properties such as the Von-Neumann entropy of quantum states without recourse to quantum tomography via a simple quantum network based on the controlled-SWAP gate. Therefore, the correspondence between the local maximal value of entanglement and local minimal value of information entropy of the stationary state may provides us a more efficient way to experimentally characterize the entanglement.

III. REMOTELY CONTROL THE DEPHASING OF TWO SOLID STATE QUBITS

In this section, we investigate how to remotely control the dephasing of two solid state qubits by making use of a preexistent multi-partite entanglement and the quantum erasing process. We assume that two solid state qubits are entangled with a remote free qubit 3, such as polarized photon. Three qubits are initially in the $|GHZ\rangle = \frac{\sqrt{2}}{2}(|11\rangle_{1,2}|H\rangle_3 + |00\rangle_{1,2}|V\rangle_3)$, where $|H\rangle$ and $|V\rangle$ are two orthogonal states of the qubit 3. If the qubit 3 is free out of any decoherence processes and one of qubit 1 and qubit 2 in a collective dephasing channel is driven by a finite-time external field, namely the evolution of qubit 1 and qubit 2 are still governed by Eq.(3), the stationary state of the three qubits can be obtained as follows:

$$\begin{aligned} \rho_{GHZ}^{(s)} = & \zeta_a\left(\frac{\Omega_1}{\gamma}, \gamma T\right)|11\rangle_{1,2}\langle 11| \otimes |H\rangle_3\langle H| \\ & + \zeta_b\left(\frac{\Omega_1}{\gamma}, \gamma T\right)|10\rangle_{1,2}\langle 10| \otimes |V\rangle_3\langle V| \\ & + \zeta_c\left(\frac{\Omega_1}{\gamma}, \gamma T\right)|01\rangle_{1,2}\langle 01| \otimes |H\rangle_3\langle H| \\ & + \zeta_d\left(\frac{\Omega_1}{\gamma}, \gamma T\right)|00\rangle_{1,2}\langle 00| \otimes |V\rangle_3\langle V| \\ & + \zeta_f\left(\frac{\Omega_1}{\gamma}, \gamma T\right)|01\rangle_{1,2}\langle 10| \otimes |H\rangle_3\langle V| \\ & + \zeta_f^*\left(\frac{\Omega_1}{\gamma}, \gamma T\right)|10\rangle_{1,2}\langle 01| \otimes |V\rangle_3\langle H|, \quad (10) \end{aligned}$$

where the coefficients ζ_i ($i = a, b, c, d, f$) can be calculated numerically according to Eq.(3). If the free qubit 3 is simply traced out, the remaining two solid state qubits 1 and 2 have not any entanglement in the stationary state. In order to maintain the entanglement of qubit 1 and qubit 2, we apply the quantum erasing process to this system. The term "quantum eraser" [10] was invented to

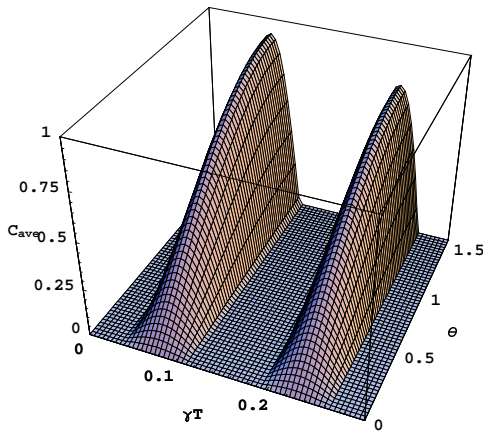


FIG. 4: The average concurrence C_{ave} of the stationary state of qubit 1 and qubit 2 after the projection measurement on qubit 3 are plotted as the function of the parameter γT and θ with $\frac{\Omega}{\gamma} = 41.25$.

describe the loss or gain of interference or, more generally quantum information, in a subensemble, based on the measurement outcomes of two complementary observables. It was reported that the implementation of two- and three-spin quantum eraser using nuclear magnetic resonance, and shown that quantum erasers provide a means of manipulating quantum entanglement [11]. Recently, we have shown how to entangle two mode thermal fields by utilizing the quantum erasing [12]. The quantum erasing process discussed here is implemented by measuring the polarizing vector of the qubit 3. The project measurement of a polarized photon has been extensively studied both in the theoretical and experimental aspects. By making a project measurement of the qubit 3 on the basis $\{\cos \frac{\theta}{2}|H\rangle + e^{i\phi} \sin \frac{\theta}{2}|V\rangle, \cos \frac{\theta}{2}|V\rangle - e^{-i\phi} \sin \frac{\theta}{2}|H\rangle\}$, the average pairwise entanglement characterized by concurrence in the reduced density matrix of qubit 1 and qubit 2 can be obtained

$$C_{ave} = 2|\sin \theta| \max(0, |\zeta_f| - \sqrt{\zeta_a \zeta_d}). \quad (11)$$

In Fig.4, we plot the average pairwise entanglement as the function of γT and θ . It is shown that an appropriate value of the parameters γT and θ can make two solid state qubits in a highly entangled state. One can also change the parameters γT and θ to control the entangle-

ment of two qubits. Since the pairwise entanglement can not arise if the qubit 3 is simply traced out, we can say that the quantum erasing process realizes the remotely control of entanglement of two solid state qubits. This may have some potential applications in remotely control of decoherence.

IV. CONCLUSIONS

In this paper, we comparatively investigate the information characterized by Von-Neumann entropy and the stationary state entanglement quantified by concurrence of two locally driven solid state qubits in the collective dephasing channel. We show that one can transform the fragile entangled states into the stationary entangled states under the collective dephasing by making use of a finite-time external driving field. The local maximal value of the stationary state concurrence corresponds to the local minimal value of information. We show how a finite-time external driving field can control the entanglement and information of the stationary state of two qubits under the collective dephasing environment. We also discuss the case that two qubits are initially in the standard Werner state. It is shown that the initial mixedness can change the stationary state entanglement and information even though the other parameters of the model are fixed. We also propose a scheme for remotely control the entanglement of two solid state qubits via multi-partite entanglement and the quantum erasing process. This idea may shed some light on the universal quantum operation based on decoherence-free qubits, and provide us a possible way to remotely suppress the detrimental effects of decoherence. Our results may have potential applications in quantum teleportation [13, 14] or other remote quantum information processes. In the future work, it may be very interesting to apply the present results to some realistic quantum information processes.

ACKNOWLEDGMENT

This project was supported by the National Natural Science Foundation of China (Project NO. 10174066).

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